



**Cambridge Assessment  
International Education**

Cambridge  
**Pre-U**



# SYLLABUS

**Cambridge International Level 3  
Pre-U Certificate in  
Further Mathematics (Principal)**

**9795**

**For centres in the UK**

For examination in 2023 and 2024

Re-sit opportunity only in 2024

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate. QN: 500/3829/1

### Changes to the syllabus for 2023 and 2024

The syllabus has been updated. The latest syllabus is version 1, published February 2020.

There are no significant changes which affect teaching.

This syllabus is available for examination in June 2023 and is being offered as a re-sit opportunity in June 2024.

**You are strongly advised to read the whole syllabus before planning your teaching programme.**

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## Introduction

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### Why choose Cambridge Pre-U?

Cambridge Pre-U is designed to equip learners with the skills required to make a success of their studies at university. Schools can choose from a wide range of subjects.

Cambridge Pre-U is built on a core set of educational aims to prepare learners for university admission, and also for success in higher education and beyond:

- to support independent and self-directed learning
- to encourage learners to think laterally, critically and creatively, and to acquire good problem-solving skills
- to promote comprehensive understanding of the subject through depth and rigour.

Cambridge Pre-U Principal Subjects are linear. A candidate must take all the components together at the end of the course in one examination series. Cambridge Pre-U Principal Subjects are assessed at the end of a two-year programme of study.

The Cambridge Pre-U nine-point grade set recognises the full range of learner ability.

### Why choose Cambridge Pre-U Further Mathematics?

- Cambridge Pre-U Further Mathematics is designed to encourage teaching and learning which enable learners to develop a positive attitude towards the subject by developing an understanding of mathematics and mathematical processes in a way that promotes confidence and enjoyment.
- Throughout this course, learners are expected to develop two parallel strands of mathematics: pure mathematics and applications of mathematics.
- The study of mathematics encourages the development of logical thought and problem-solving skills.
- The linear assessment structure means that learners are tested at the end of the two-year course. This allows learners to approach the examination in a mature and confident way, with time to develop their knowledge, understanding and skills.
- Cambridge Pre-U Further Mathematics involves the acquisition of skills that can be applied in a wide range of contexts.
- Cambridge Pre-U Further Mathematics extends learners' range of mathematical skills and techniques, building on the foundation of Cambridge Pre-U Mathematics as a preparation for further study.

### Prior learning

Cambridge Pre-U builds on the knowledge, understanding and skills gained by learners achieving a good pass in Level 1/Level 2 qualifications in mathematics or related subjects.

Candidates should have studied or be studying Cambridge Pre-U Mathematics. Knowledge of the syllabus content of Cambridge Pre-U Mathematics is assumed for both components of Cambridge Pre-U Further Mathematics.

## Progression

Cambridge Pre-U is considered to be an excellent preparation for university, employment and life. It helps to develop the in-depth subject knowledge and understanding which are so important to universities and employers. While it is a satisfying subject in its own right, mathematics is also a prerequisite for further study in an increasing range of subjects. For this reason, learners following this course will be expected to apply their mathematical knowledge in the contexts of both mechanics and probability, and will also be presented with less familiar scenarios. This syllabus provides a sound foundation in mathematics for higher education courses or other career pathways where mathematics is used extensively.

The qualification is regulated at Level 3 of the Regulated Qualifications Framework and provides a solid grounding for candidates to pursue a variety of progression pathways.

## Cambridge Pre-U Diploma

If learners choose, they can combine Cambridge Pre-U qualifications to achieve the Cambridge Pre-U Diploma; this comprises three Cambridge Pre-U Principal Subjects\* together with Global Perspectives and Independent Research (GPR). The Cambridge Pre-U Diploma, therefore, provides the opportunity for interdisciplinary study informed by an international perspective and includes an independent research project.

first year	second year
<b>CAMBRIDGE PRE-U DIPLOMA</b>	
Cambridge Pre-U Principal Subject	
Cambridge Pre-U Principal Subject*	
Cambridge Pre-U Principal Subject*	
Cambridge Pre-U Global Perspectives and Independent Research (GPR)	

\* Up to two A Levels, Scottish Advanced Highers or IB Diploma programme courses at higher level can be substituted for Principal Subjects.

Learn more about the Cambridge Pre-U Diploma at [www.cambridgeinternational.org/cambridgepreu](http://www.cambridgeinternational.org/cambridgepreu)

## Support

Cambridge International provides a wide range of support for Pre-U syllabuses, which includes recommended resource lists, Teacher Guides and Example Candidate Response booklets. Teachers can access these support materials at the School Support Hub [www.cambridgeinternational.org/support](http://www.cambridgeinternational.org/support)

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## Syllabus aims

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The aims of the syllabus, listed below, are the same for all candidates and are to:

- enable learners to develop a range of mathematical skills and techniques, appreciating their applications in a wide range of contexts, and to apply these techniques to problem-solving in familiar and less familiar contexts
- enable learners to develop an understanding of how different branches of mathematics are connected
- enable learners to recognise how a situation may be represented mathematically and understand how mathematical models can be refined
- encourage learners to use mathematics as an effective means of communication, through the use of correct mathematical language and notation and through the construction of sustained logical arguments, including an appreciation of the limitations of calculator use in relation to obtaining exact solutions.

Cambridge Assessment International Education is an education organisation and politically neutral. The contents of this syllabus, examination papers and associated materials do not endorse any political view. We endeavour to treat all aspects of the exam process neutrally.



## Scheme of assessment

For Cambridge Pre-U Further Mathematics, candidates take both components.

Component		Weighting
<b>Paper 1 Further Pure Mathematics</b>	<b>3 hours</b>	50%
Written paper, with structured questions, externally assessed, 120 marks		
<b>Paper 2 Further Applications of Mathematics</b>	<b>3 hours</b>	50%
Written paper, with structured questions, externally assessed, 120 marks		

### Availability

This syllabus is examined in the June 2023 series and is being offered in the June 2024 series as a re-sit opportunity only. This syllabus will not be offered again and there will be no further re-sit opportunities.

This syllabus is available to private candidates.

### Combining this with other syllabuses

Candidates can combine this syllabus in a series with any other Cambridge International syllabus, except syllabuses with the same title at the same level.

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## Assessment objectives

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<b>A01</b>	Manipulate mathematical expressions accurately, round answers to an appropriate degree of accuracy, and understand the limitations of solutions obtained using calculators.
<b>A02</b>	Construct rigorous mathematical arguments and proofs through the use of precise statements and logical deduction, including extended arguments for problems presented in unstructured form.
<b>A03</b>	Recall, select and apply knowledge of mathematical facts, concepts and techniques in a variety of contexts.
<b>A04</b>	Understand how mathematics can be used to model situations in the real world and solve problems in relation to both standard models and less familiar contexts, interpreting the results.



## Relationship between scheme of assessment and assessment objectives

The approximate weightings allocated to each of the assessment objectives (AOs) are summarised below.

### Assessment objectives as a percentage of the qualification

Assessment objective	Weighting in Pre-U %
<b>A01</b>	32 ± 3
<b>A02</b>	16 ± 3
<b>A03</b>	37 ± 3
<b>A04</b>	18 ± 2

### Assessment objectives as a percentage of each component

Assessment objective	Weighting in components %	
	Paper 1	Paper 2
<b>A01</b>	36 ± 3	28 ± 3
<b>A02</b>	20 ± 3	12 ± 3
<b>A03</b>	47 ± 3	27 ± 3
<b>A04</b>	0	36 ± 4

## Grading and reporting

Cambridge International Level 3 Pre-U Certificates (Principal Subjects and Global Perspectives Short Course) are qualifications in their own right. Cambridge Pre-U reports achievement on a scale of nine grades: Distinction 1, Distinction 2, Distinction 3, Merit 1, Merit 2, Merit 3, Pass 1, Pass 2 and Pass 3.

Cambridge Pre-U band	Cambridge Pre-U grade
Distinction	1
	2
	3
Merit	1
	2
	3
Pass	1
	2
	3

## Grade descriptions

Grade descriptions are provided to give an indication of the standards of achievement likely to have been shown by candidates awarded particular grades. Weakness in one aspect of the examination may be balanced by a better performance in some other aspect.

The following grade descriptions indicate the level of attainment characteristic of the middle of the given grade band.

### Distinction (D2)

- Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill.
- If errors are made in candidates' calculations or logic, these are mostly noticed and corrected.
- Candidates make appropriate and efficient use of calculators and other permitted resources, and are aware of any limitations to their use.
- Candidates present results to an appropriate degree of accuracy.
- Candidates use mathematical language correctly and proceed logically and rigorously through extended arguments or proofs.
- When confronted with unstructured problems, candidates can mostly devise and implement an effective solution strategy.
- Candidates recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts.
- Candidates are usually able to solve problems in less familiar contexts.
- Candidates correctly refer results from calculations using a mathematical model to the original situation; they give sensible interpretations of their results in context and mostly make sensible comments or predictions.
- Candidates recall or recognise almost all the standard models that are needed, and select appropriate ones to represent a wide variety of situations in the real world.
- Candidates comprehend or understand the meaning of almost all translations into mathematics of common realistic contexts.
- Candidates make intelligent comments on any modelling assumptions.

### Merit (M2)

- Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill.
- Candidates often notice and correct errors in their calculations.
- Candidates usually make appropriate and efficient use of calculators and other permitted resources, and are often aware of any limitations to their use.
- Candidates usually present results to an appropriate degree of accuracy.
- Candidates use mathematical language with some skill and usually proceed logically through extended arguments or proofs.
- When confronted with unstructured problems, candidates usually devise and implement an effective solution strategy.
- Candidates recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts.
- Candidates are often able to solve problems in less familiar contexts.

- Candidates usually correctly refer results from calculations using a mathematical model to the original situation; they usually give sensible interpretations of their results in context and sometimes make sensible comments or predictions.
- Candidates recall or recognise most of the standard models that are needed, and usually select appropriate ones to represent a variety of situations in the real world.
- Candidates comprehend or understand the meaning of most translations into mathematics of common realistic contexts.
- Candidates often make intelligent comments on any modelling assumptions.

### Pass (P2)

- Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill.
- If errors are made in candidates' calculations or logic, these are sometimes noticed and corrected.
- Candidates usually make appropriate and efficient use of calculators and other permitted resources, and are often aware of any limitations to their use.
- Candidates often present results to an appropriate degree of accuracy.
- Candidates frequently use mathematical language correctly and occasionally proceed logically through extended arguments or proofs.
- When confronted with unstructured problems, candidates can sometimes make some headway with an effective solution strategy.
- Candidates recall or recognise some of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use in some contexts.
- Candidates try to solve problems in less familiar contexts.
- Candidates frequently correctly refer results from calculations using a mathematical model to the original situation; they sometimes interpret their results in context and attempt to make sensible comments or predictions.
- Candidates recall or recognise some of the standard models that are needed, and frequently select appropriate ones to represent a variety of situations in the real world.
- Candidates frequently comprehend or understand the meaning of translations into mathematics of common realistic contexts.

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## Description of components

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For both components, knowledge of the content of the Cambridge Pre-U Mathematics syllabus is assumed.

### Paper 1 Further Pure Mathematics

Written paper, 3 hours, 120 marks

- rational functions
- roots of polynomial equations
- complex numbers
- de Moivre's theorem
- polar coordinates
- summation of series
- mathematical induction
- calculus
- hyperbolic functions
- differential equations
- vector geometry
- matrices
- groups

The written paper will consist of a mixture of short, medium and longer questions, with a total of 120 marks. In addition to the topics listed, candidates will be expected to apply their knowledge of pure mathematics to questions set in less familiar contexts. Candidates will be expected to answer all questions.

## Paper 2 Further Applications of Mathematics

Written paper, 3 hours, 120 marks

- Probability
  - Poisson distribution
  - normal distribution as an approximation
  - continuous random variables
  - linear combinations of random variables
  - estimation
  - probability generating functions
  - moment generating functions
- Mechanics
  - energy, work and power
  - motion in a circle
  - equilibrium of a rigid body
  - elastic strings and springs
  - simple harmonic motion (SHM)
  - further particle dynamics
  - linear motion under a variable force

The written paper will consist of a mixture of short, medium and longer questions, with a total of 120 marks, with approximately equal marks for mechanics and probability. In addition to the topics listed, candidates will be expected to apply their knowledge of pure mathematics to questions set in less familiar contexts. Candidates will be expected to answer all questions.

### Use of calculators

The use of scientific calculators will be permitted in all papers. Graphic calculators will not be permitted. Candidates will be expected to be aware of the limitations inherent in the use of calculators.

### Mathematical tables and formulae

Candidates will be provided with a booklet of mathematical formulae and tables for use in the examination.

## Syllabus content

This syllabus gives you the flexibility to design a course that will interest, challenge and engage your learners. Where appropriate you are responsible for selecting suitable subject contexts, resources and examples to support your learners' study. These should be appropriate for the learners' age, cultural background and learning context as well as complying with your school policies.

### Paper 1 Further Pure Mathematics

#### Rational functions

Candidates should be able to:

- sketch graphs of simple rational functions, including the determination of oblique asymptotes, in cases where the degree of the numerator and the denominator are at most 2 (detailed plotting of curves will not be expected, but sketches will generally be expected to show significant features, such as turning points, asymptotes and intersections with axes; the use of a discriminant in determining the set of values taken by the function is included).

#### Roots of polynomial equations

Candidates should be able to:

- use the relations between the symmetric functions of the roots of polynomial equations and the coefficients, for equations up to degree 4
- use a substitution to obtain an equation whose roots are related in a simple way to those of the original equation.

#### Complex numbers

Candidates should be able to:

- illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram, for example  $|z - a| < k$ ,  $|z - a| = |z - b|$  and  $\arg(z - a) = \alpha$ , and combinations of such forms
- find square roots of complex numbers in cartesian form by forming and solving appropriate quadratic equations
- recognise the modulus-argument form of a complex number, and convert a complex number in this form into cartesian form, and vice versa
- multiply and divide complex numbers in modulus-argument form, and calculate integral powers of complex numbers
- represent a complex number as  $re^{i\theta}$
- use the relation  $e^{i\theta} = \cos \theta + i \sin \theta$ .

## De Moivre's theorem

Candidates should be able to:

- understand de Moivre's theorem, for positive and negative integer exponents, in terms of the geometrical effect of multiplication and division of complex numbers
- use de Moivre's theorem to express trigonometric ratios of multiple angles in terms of powers of trigonometric ratios of the fundamental angle
- use expressions for  $\sin \theta$  and  $\cos \theta$  in terms of  $e^{i\theta}$ , e.g. for expressing powers of  $\sin \theta$  and  $\cos \theta$  in terms of multiple angles or for summing series
- find and use the  $n$ th roots of complex numbers.

## Polar coordinates

Candidates should be able to:

- understand the relations between cartesian and polar coordinates (using the convention  $r \geq 0$ ), and convert equations of curves from cartesian to polar form and vice versa
- sketch simple polar curves for  $0 \leq \theta < 2\pi$  or  $-\pi < \theta \leq \pi$  or a subset of these intervals (detailed plotting of curves will not be expected, but sketches will generally be expected to show significant features, such as symmetry, the form of the curve at the pole and least/greatest values of  $r$ )
- use the formula  $\frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$  for the area of a sector in simple cases.

## Summation of series

Candidates should be able to:

- use the standard results for  $\Sigma r$ ,  $\Sigma r^2$  and  $\Sigma r^3$  to find related sums
- use the method of differences to obtain the sum of a finite series, e.g. by expressing the general term in partial fractions
- recognise, by direct consideration of a sum to  $n$  terms, when a series is convergent, and find the sum to infinity in such cases.

## Mathematical induction

Candidates should be able to:

- use the method of mathematical induction to establish a given result (questions set may involve sums of series, divisibility or inequalities, for example)
- recognise situations where conjecture based on a limited trial followed by inductive proof is a useful strategy, and carry this out in simple cases, e.g. to find the  $n$ th derivative of  $xe^x$ .

## Calculus

Candidates should be able to:

- derive and use reduction formulae for the evaluation of definite integrals in simple cases
- differentiate inverse trigonometric functions
- recognise integrals of functions of the form  $\frac{1}{\sqrt{a^2 - x^2}}$  and  $\frac{1}{a^2 + x^2}$ , and be able to integrate associated functions by using trigonometric substitutions
- derive and use the Maclaurin and the Taylor series
- calculate the arc length of a curve using cartesian, parametric or polar forms
- calculate the area of a surface of revolution about either the  $x$ - or  $y$ -axis using cartesian, parametric or polar forms.



## Hyperbolic functions

Candidates should be able to:

- understand the definitions of the hyperbolic functions  $\sinh x$ ,  $\cosh x$ ,  $\tanh x$ ,  $\operatorname{sech} x$ ,  $\operatorname{cosech} x$  and  $\operatorname{coth} x$ , in terms of the exponential function
- sketch the graphs of hyperbolic functions
- use identities involving hyperbolic functions
- differentiate and integrate hyperbolic functions
- understand and use the definitions of the inverse hyperbolic functions, and derive and use their logarithmic forms
- recognise integrals of functions of the form  $\frac{1}{\sqrt{a^2 + x^2}}$  and  $\frac{1}{\sqrt{x^2 - a^2}}$ , and integrate associated functions by using hyperbolic substitutions.

## Differential equations

Candidates should be able to:

- find an integrating factor for a first-order linear differential equation, and use an integrating factor to find the general solution
- use a given substitution to reduce a first- or second-order equation to a more standard form
- recall the meaning of the terms complementary function and particular integral in the context of linear differential equations, and use the fact that the general solution is the sum of the complementary function and a particular integral
- find the complementary function for a first- or second-order linear differential equation with constant coefficients
- recall the form of, and find, a particular integral for a first- or second-order linear differential equation in the cases where  $ax^2 + bx + c$  or  $ae^{bx}$  or  $a \cos px + b \sin px$  is a suitable form, and in other cases find the appropriate coefficient(s) given a suitable form of particular integral
- use initial conditions to find a particular solution to a differential equation, and interpret the solution in the context of a problem modelled by the differential equation
- use the Taylor series method to obtain series solutions to differential equations.

## Vector geometry

Candidates should be able to:

- calculate the vector product of two vectors
- understand and use the equation of a plane expressed in any of the forms  $(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0$  or  $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b} + \mu \mathbf{c}$  or  $ax + by + cz = d$
- find the line of intersection of two planes
- calculate the shortest distance of a point from a line or from a plane
- calculate the scalar triple product, and use it to find the volume of a parallelepiped or a tetrahedron
- calculate the shortest distance between two skew lines.

## Matrices

Candidates should be able to:

- carry out operations of matrix addition, subtraction and multiplication, and recognise the terms null (or zero) matrix and identity (or unit) matrix
- recall the meaning of the terms *singular* and *non-singular* as applied to square matrices and, for  $2 \times 2$  and  $3 \times 3$  matrices, evaluate determinants and find inverses of non-singular matrices
- understand and use the result, for non-singular matrices, that  $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$
- understand the use of  $2 \times 2$  matrices to represent certain geometrical transformations in the  $x$ - $y$  plane, and in particular:
  - recognise that the matrix product  $\mathbf{AB}$  represents the transformation that results from the transformation represented by  $\mathbf{B}$  followed by the transformation represented by  $\mathbf{A}$
  - recall how the area scale factor of a transformation is related to the determinant of the corresponding matrix
  - find the matrix that represents a given transformation or sequence of transformations (understanding of the terms *rotation*, *reflection*, *enlargement*, *stretch* and *shear* will be required)
- formulate a problem involving the solution of two linear simultaneous equations in two unknowns, or three equations in three unknowns, as a problem involving the solution of a matrix equation, and vice versa
- understand the cases that may arise concerning the consistency or inconsistency of two or three linear simultaneous equations, relate them to the singularity or otherwise of the corresponding square matrix, and solve consistent systems.

## Groups

Candidates should be able to:

- recall that a group consists of a set of elements together with a binary operation which is closed and associative, for which an identity exists in the set, and for which every element has an inverse in the set
- use the basic group properties to show that a given structure is, or is not, a group (questions may be set on, for example, groups of matrices, transformations, permutations, integers modulo  $n$ )
- use algebraic methods to establish properties in abstract groups in easy cases, e.g. to show that any group in which every element is self-inverse is commutative (abelian)
- recall the meaning of the term *order* as applied both to groups and to elements of a group, and determine the order of elements in a given group
- understand the idea of a subgroup of a given group, find subgroups in simple cases, and show that given subsets are, or are not, (proper) subgroups
- recall and apply Lagrange's theorem concerning the order of a subgroup of a finite group (proof of the theorem is not required)
- recall the meaning of the term *cyclic* as applied to groups, and show familiarity with the structure of finite groups up to order 7 (questions on groups of higher order are not excluded, but no particular prior knowledge of such groups is expected)
- understand the idea of isomorphism between groups, and determine whether given finite groups are, or are not, isomorphic.

## Paper 2 Further Applications of Mathematics

### PROBABILITY

#### Poisson distribution

Candidates should be able to:

- calculate probabilities for the distribution  $Po(\lambda)$ , both directly from the formula and also by using tables of cumulative Poisson probabilities
- use the result that if  $X \sim Po(\lambda)$ , then the mean and variance of  $X$  are each equal to  $\lambda$
- understand informally the relevance of the Poisson distribution to the distribution of random events, and use the Poisson distribution as a model
- use the Poisson distribution as an approximation to the binomial distribution where appropriate.

#### Normal distribution as an approximation

Candidates should be able to:

- recall conditions under which the normal distribution can be used as an approximation to the binomial distribution, and use this approximation, with a continuity correction, in solving problems
- use the normal distribution, with a continuity correction, as an approximation to the Poisson distribution, where appropriate.

#### Continuous random variables

Candidates should be able to:

- understand the concept of a continuous random variable, and recall and use properties of a probability density function (P.D.F.) (defined over a single interval or piecewise)
- use a probability density function to solve problems involving probabilities, and to calculate the mean and variance of a continuous random variable
- use, in simple cases, the general result  $E(g(X)) = \int g(x)f(x)dx$ , where  $f(x)$  is the probability density function of the continuous random variable  $X$ , and  $g(X)$  is a function of  $X$
- understand and use the relationship between the probability density function and the cumulative distribution function (C.D.F.), and use either to evaluate the median, quartiles and other percentiles
- use cumulative distribution functions of related variables in simple cases, e.g. given the C.D.F. of a variable  $X$ , find the C.D.F. and hence the P.D.F. of  $Y$ , where  $Y = X^3$ .

## Linear combinations of random variables

Candidates should be able to:

- use, in the course of solving problems, the results that:
  - $E(aX + b) = aE(X) + b$  and  $\text{Var}(aX + b) = a^2 \text{Var}(X)$
  - $E(aX + bY) = aE(X) + bE(Y)$
  - $\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y)$  for independent  $X$  and  $Y$
  - if  $X$  has a normal distribution, then so does  $aX + b$
  - if  $X$  and  $Y$  are independent random variables with normal distributions, then  $aX + bY$  has a normal distribution
  - if  $X$  and  $Y$  are independent random variables with Poisson distributions, then  $X + Y$  has a Poisson distribution
- recognise that the sample mean  $\bar{X}$  has a normal distribution if  $X$  is normally distributed, and has an approximately normal distribution in other cases for sufficiently large sample size (Central Limit Theorem), and use the results  $E(\bar{X}) = \mu$ ,  $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$ .

## Estimation

Candidates should be able to:

- understand the distinction between a sample and a population, and appreciate the benefits of randomness in choosing samples
- understand that an estimator of a parameter of a distribution is a random variable, and understand the meaning of biased and unbiased estimators (both discrete and continuous distributions are included)
- determine whether a given estimator is biased or unbiased, and construct estimators in simple cases
- calculate unbiased estimates of the population mean and variance from a random sample, using either raw or summarised data
- determine a confidence interval for a population mean, using a normal distribution, in the context of:
  - a random sample drawn from a normal population of known variance
  - a large random sample, using the Central Limit Theorem, and an unbiased estimate of the variance of the population derived from the sample
- determine, from a large random sample, an approximate confidence interval for a population proportion
- use a  $t$ -distribution with the appropriate number of degrees of freedom, in the context of a small random sample drawn from a normal population of unknown variance, to determine a confidence interval for the population mean
- calculate a pooled estimate of a population variance based on the data from two random samples
- calculate a confidence interval for a difference of population means, using a normal distribution or a  $t$ -distribution, as appropriate
- comment on assumptions that have been made, for example the randomness or normality of a distribution, and comment critically on statements in the light of the confidence interval that has been calculated.

## Probability generating functions

Candidates should be able to:

- understand the concept of a probability generating function (P.G.F.), and construct and use the probability generating function for given distributions (including discrete uniform, binomial, geometric and Poisson)
- use formulae for the mean and variance of a discrete random variable in terms of its probability generating function, and use these formulae to calculate the mean and variance of probability distributions
- use the result that the probability generating function of the sum of independent random variables is the product of the probability generating functions of those random variables.

## Moment generating functions

Candidates should be able to:

- understand the concept of a moment generating function (M.G.F.) for both discrete and continuous random variables; construct and use the moment generating function for given distributions
- use the moment generating function of a given distribution to find the mean and variance
- use the result that the moment generating function of the sum of independent random variables is the product of the moment generating functions of those random variables.

## MECHANICS

### Energy, work and power

Candidates should be able to:

- understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force
- understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae
- understand and use the relationship between the change in energy of a system and the work done by the external forces, and use, in appropriate cases, the principle of conservation of energy
- use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion.

### Motion in a circle

Candidates should be able to:

- understand the concept of angular speed for a particle moving in a circle, and use the relation  $v = r\omega$
- understand that the acceleration of a particle moving in a circle with constant speed is directed towards the centre of the circle, and use the formulae  $a = r\omega^2$  and  $a = \frac{v^2}{r}$
- solve problems which can be modelled by the motion of a particle moving in a horizontal circle with constant speed
- use formulae for the radial and transverse components of acceleration for a particle moving in a circle with variable speed
- solve problems which can be modelled as a particle, or a pair of connected particles, moving without loss of energy in a vertical circle (including the determination of points where circular motion breaks down, e.g. through loss of contact with a surface or a string becoming slack).

## Equilibrium of a rigid body

Candidates should be able to:

- understand and use the result that the effect of gravity on a rigid body is equivalent to a single force acting at the centre of mass of the body, and identify the centre of mass by considerations of symmetry in suitable cases
- calculate the moment of a force about a point in two-dimensional situations only (understanding of the vector nature of moments is not required)
- use the principle that, under the action of coplanar forces, a rigid body is in equilibrium if and only if (i) the vector sum of the forces is zero, and (ii) the sum of the moments of the forces about any point is zero
- solve problems involving the equilibrium of a single rigid body under the action of coplanar forces (problems set will not involve complicated trigonometry).

## Elastic strings and springs

Candidates should be able to:

- use Hooke's law as a model relating the force in an elastic string or spring to the extension or compression, and understand the term *modulus of elasticity*
- use the formula for the elastic potential energy stored in a string or spring
- solve problems involving forces due to elastic strings or springs, including those where considerations of work and energy are needed.

## Simple harmonic motion (SHM)

Candidates should be able to:

- recall a definition of SHM, understand the concepts of period and amplitude, and use standard SHM formulae in solving problems
- set up the differential equation of motion in problems leading to SHM, quote appropriate forms of solution, and identify the period and amplitude of the motion
- use Newton's second law, together with the approximation  $\sin \theta \approx \theta$ , to show that small oscillations of a simple pendulum may be modelled as SHM; solve associated problems and understand the limitations of this model.

## Further particle dynamics

Candidates should be able to:

- understand the vector nature of impulse and momentum for motion in two dimensions
- solve problems that may be modelled as the oblique impact of two smooth spheres or as the oblique impact of a smooth sphere with a fixed surface
- solve harder problems involving projectile motion, for example projectiles launched and landing on an inclined plane and points accessible to the projectile.

## Linear motion under a variable force

Candidates should be able to:

- use  $\frac{dx}{dt}$  for velocity, and  $\frac{dv}{dt}$  or  $v \frac{dv}{dx}$  for acceleration, as appropriate
- solve problems which can be modelled as the linear motion of a particle under the action of a variable force, by setting up and solving an appropriate differential equation.

## Mathematical formulae and statistical tables

Examinations for this syllabus will use relevant formulae from the following list.

### PURE MATHEMATICS

#### Mensuration

$$\text{Surface area of sphere} = 4\pi r^2$$

$$\text{Area of curved surface of cone} = \pi r \times \text{slant height}$$

#### Trigonometry

$$a^2 = b^2 + c^2 - 2bc \cos A$$

#### Arithmetic series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n - 1)d\}$$

#### Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

#### Summations

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$

#### Binomial series

$$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n, (n \in \mathbb{N}), \text{ where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \cdot 2}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \cdot 2 \dots r}x^r + \dots (|x| < 1, n \in \mathbb{R})$$

#### Logarithms and exponentials

$$e^{x \ln a} = a^x$$

#### Complex numbers

$$\{r(\cos \theta + i \sin \theta)\}^n = r^n (\cos n\theta + i \sin n\theta)$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

The roots of  $z^n = 1$  are given by  $z = e^{\frac{2\pi ki}{n}}$ , for  $k = 0, 1, 2, \dots, n - 1$

**Maclaurin's series**

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \dots + \frac{x^r}{r!}f^{(r)}(0) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 \leq x \leq 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 < x < 1)$$

**Hyperbolic functions**

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln\{x + \sqrt{x^2 - 1}\} \quad (x \geq 1)$$

$$\sinh^{-1} x = \ln\{x + \sqrt{x^2 + 1}\}$$

$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$$

**Coordinate geometry**

The perpendicular distance from  $(h, k)$  to  $ax + by + c = 0$  is  $\frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$

The acute angle between lines with gradients  $m_1$  and  $m_2$  is  $\tan^{-1} \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\text{For } t = \tan \frac{1}{2}A: \sin A = \frac{2t}{1+t^2}, \cos A = \frac{1-t^2}{1+t^2}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$



## Vectors

The resolved part of  $\mathbf{a}$  in the direction of  $\mathbf{b}$  is  $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$

The point dividing  $AB$  in the ratio  $\lambda:\mu$  is  $\frac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$

$$\text{Vector product: } \mathbf{a} \times \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \sin \theta \hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

If  $A$  is the point with position vector  $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$  and the direction vector  $\mathbf{b}$  is given by

$\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ , then the straight line through  $A$  with direction vector  $\mathbf{b}$  has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \quad (= \lambda)$$

The plane through  $A$  with normal vector  $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$  has cartesian equation

$$n_1x + n_2y + n_3z + d = 0 \quad \text{where } d = -\mathbf{a} \cdot \mathbf{n}$$

The plane through non-collinear points  $A, B$  and  $C$  has vector equation

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

The plane through the point with position vector  $\mathbf{a}$  and parallel to  $\mathbf{b}$  and  $\mathbf{c}$  has equation  $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

The perpendicular distance of  $(\alpha, \beta, \gamma)$  from  $n_1x + n_2y + n_3z + d = 0$  is  $\frac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

## Matrix transformations

Anticlockwise rotation through  $\theta$  about  $O$ :  $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Reflection in the line  $y = (\tan \theta)x$ :  $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

## Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$

**Integration** (+ constant;  $a > 0$  where relevant)

$f(x)$	$\int f(x) dx$
$\sec^2 kx$	$\frac{1}{k} \tan kx$
$\tan x$	$\ln \sec x $
$\cot x$	$\ln \sin x $
$\operatorname{cosec} x$	$-\ln \operatorname{cosec} x + \cot x  = \ln \tan(\frac{1}{2}x) $
$\sec x$	$\ln \sec x + \tan x  = \ln \tan(\frac{1}{2}x + \frac{1}{4}\pi) $
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \cosh x$
$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}(\frac{x}{a}) \quad ( x  < a)$
$\frac{1}{a^2 + x^2}$	$\frac{1}{a} \tan^{-1}(\frac{x}{a})$
$\frac{1}{\sqrt{x^2 - a^2}}$	$\cosh^{-1}(\frac{x}{a})$ or $\ln\{x + \sqrt{x^2 - a^2}\} \quad (x > a)$
$\frac{1}{\sqrt{a^2 + x^2}}$	$\sinh^{-1}(\frac{x}{a})$ or $\ln\{x + \sqrt{x^2 + a^2}\}$
$\frac{1}{a^2 - x^2}$	$\frac{1}{2a} \ln \frac{a+x}{a-x}  = \frac{1}{a} \tanh^{-1}(\frac{x}{a}) \quad ( x  < a)$
$\frac{1}{x^2 - a^2}$	$\frac{1}{2a} \ln \frac{x-a}{x+a} $
$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$	

**Area of a sector**

$$A = \frac{1}{2} \int r^2 d\theta \quad (\text{polar coordinates})$$

$$A = \frac{1}{2} \int \left( x \frac{dy}{dt} - y \frac{dx}{dt} \right) dt \quad (\text{parametric form})$$

**Arc length**

$$s = \int \sqrt{1 + \left( \frac{dy}{dx} \right)^2} dx \quad (\text{cartesian coordinates})$$

$$s = \int \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} dt \quad (\text{parametric form})$$

$$s = \int \sqrt{r^2 + \left( \frac{dr}{d\theta} \right)^2} d\theta \quad (\text{polar form})$$

**Surface area of revolution**

$$S_x = 2\pi \int y ds$$

$$S_y = 2\pi \int x ds$$

### Numerical solution of equations

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

## MECHANICS

### Motion in a circle

Transverse velocity:  $v = r\dot{\theta}$

Transverse acceleration:  $\dot{v} = v\ddot{\theta}$

Radial acceleration:  $-r\dot{\theta}^2 = -\frac{v^2}{r}$

## PROBABILITY

### Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A)$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$$

$$\text{Bayes' Theorem: } P(A_j|B) = \frac{P(A_j)P(B|A_j)}{\sum P(A_i)P(B|A_i)}$$

### Discrete distributions

For a discrete random variable  $X$  taking values  $x_i$  with probabilities  $p_i$

$$\text{Expectation (mean): } E(X) = \mu = \sum x_i p_i$$

$$\text{Variance: } \text{Var}(X) = \sigma^2 = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$$

$$\text{For a function } g(X): E(g(X)) = \sum g(x_i) p_i$$

The probability generating function (P.G.F.) of  $X$  is  $G_X(t) = E(t^X)$ ,

$$\text{and } E(X) = G'_X(1), \text{ Var}(X) = G''_X(1) + G'_X(1) - \{G'_X(1)\}^2$$

For  $Z = X + Y$ , where  $X$  and  $Y$  are independent:  $G_Z(t) = G_X(t)G_Y(t)$

The moment generating function (M.G.F.) of  $X$  is  $M_X(t) = E(e^{tX})$ ,

$$\text{and } E(X) = M'_X(0), E(X^n) = M_X^{(n)}(0), \text{ Var}(X) = M''_X(0) - \{M'_X(0)\}^2$$

For  $Z = X + Y$ , where  $X$  and  $Y$  are independent:  $M_Z(t) = M_X(t)M_Y(t)$

### Standard discrete distributions

Distribution of $X$	$P(X=x)$	Mean	Variance	P.G.F.	M.G.F.
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	$np$	$np(1-p)$	$(1-p+pt)^n$	$(1-p+pe^t)^n$
Poisson $Po(\lambda)$	$e^{-\lambda} \frac{\lambda^x}{x!}$	$\lambda$	$\lambda$	$e^{\lambda(t-1)}$	$e^{\lambda(e^t-1)}$
Geometric $Geo(p)$ on $1, 2, \dots$	$p(1-p)^{x-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pt}{1-(1-p)t}$	$\frac{pe^t}{1-(1-p)e^t}$

### Continuous distributions

For a continuous random variable  $X$  having probability density function (P.D.F.)  $f$

$$\text{Expectation (mean): } E(X) = \mu = \int x f(x) dx$$

$$\text{Variance: } \text{Var}(X) = \sigma^2 = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2$$

$$\text{For a function } g(X) : E(g(X)) = \int g(x) f(x) dx$$

$$\text{Cumulative distribution function: } F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$$

The moment generating function (M.G.F.) of  $X$  is  $M_X(t) = E(e^{tX})$ ,

$$\text{and } E(X) = M'_X(0), E(X^n) = M_X^{(n)}(0), \text{Var}(X) = M''_X(0) - \{M'_X(0)\}^2$$

For  $Z = X + Y$ , where  $X$  and  $Y$  are independent:  $M_Z(t) = M_X(t)M_Y(t)$

### Standard continuous distributions

Distribution of $X$	P.D.F.	Mean	Variance	M.G.F.
Uniform (Rectangular) on $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$	$\frac{e^{bt} - e^{at}}{(b-a)t}$
Exponential	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - t}$
Normal $N(\mu, \sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	$\mu$	$\sigma^2$	$e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

### Expectation algebra

For independent random variables  $X$  and  $Y$

$$E(XY) = E(X)E(Y), \text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$$

### Sampling distributions

For a random sample  $X_1, X_2, \dots, X_n$  of  $n$  independent observations from a distribution having mean  $\mu$  and variance  $\sigma^2$

$$\bar{X} \text{ is an unbiased estimator of } \mu, \text{ with } \text{Var}(\bar{X}) = \frac{\sigma^2}{n}$$

$$S^2 \text{ is an unbiased estimator of } \sigma^2, \text{ where } S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1}$$

For a random sample of  $n$  observations from  $N(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

$$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1} \text{ (also valid in matched-pairs situations)}$$

If  $X$  is the observed number of successes in  $n$  independent Bernoulli trials, in each of which the probability of

success is  $p$ , and  $Y = \frac{X}{n}$ , then  $E(Y) = p$  and  $\text{Var}(Y) = \frac{p(1-p)}{n}$

For a random sample of  $n_x$  observations from  $N(\mu_x, \sigma_x^2)$  and, independently, a random sample of  $n_y$  observations from  $N(\mu_y, \sigma_y^2)$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0, 1)$$

If  $\sigma_x^2 = \sigma_y^2 = \sigma^2$  (unknown), then  $\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2 \left(\frac{1}{n_x} + \frac{1}{n_y}\right)}} \sim t_{n_x + n_y - 2}$ , where  $S_p^2 = \frac{(n_x - 1)S_x^2 + (n_y - 1)S_y^2}{n_x + n_y - 2}$

### Correlation and regression

For a set of  $n$  pairs of values  $(x_i, y_i)$

$$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$$

$$S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{(\sum y_i)^2}{n}$$

$$S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}$$

The product-moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\left\{ \sum (x_i - \bar{x})^2 \right\} \left\{ \sum (y_i - \bar{y})^2 \right\}}} = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sqrt{\left( \sum x_i^2 - \frac{(\sum x_i)^2}{n} \right) \left( \sum y_i^2 - \frac{(\sum y_i)^2}{n} \right)}}$$

The regression coefficient of  $y$  on  $x$  is  $b = \frac{S_{xy}}{S_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$

Least squares regression line of  $y$  on  $x$  is  $y = a + bx$  where  $a = \bar{y} - b\bar{x}$

















### CUMULATIVE POISSON PROBABILITIES

$\lambda$	4.00	4.10	4.20	4.30	4.40	4.50	4.60	4.70	4.80	4.90
$x = 0$	0.0183	0.0166	0.0150	0.0136	0.0123	0.0111	0.0101	0.0091	0.0082	0.0074
1	0.0916	0.0845	0.0780	0.0719	0.0663	0.0611	0.0563	0.0518	0.0477	0.0439
2	0.2381	0.2238	0.2102	0.1974	0.1851	0.1736	0.1626	0.1523	0.1425	0.1333
3	0.4335	0.4142	0.3954	0.3772	0.3594	0.3423	0.3257	0.3097	0.2942	0.2793
4	0.6288	0.6093	0.5898	0.5704	0.5512	0.5321	0.5132	0.4946	0.4763	0.4582
5	0.7851	0.7693	0.7531	0.7367	0.7199	0.7029	0.6858	0.6684	0.6510	0.6335
6	0.8893	0.8786	0.8675	0.8558	0.8436	0.8311	0.8180	0.8046	0.7908	0.7767
7	0.9489	0.9427	0.9361	0.9290	0.9214	0.9134	0.9049	0.8960	0.8867	0.8769
8	0.9786	0.9755	0.9721	0.9683	0.9642	0.9597	0.9549	0.9497	0.9442	0.9382
9	0.9919	0.9905	0.9889	0.9871	0.9851	0.9829	0.9805	0.9778	0.9749	0.9717
10	0.9972	0.9966	0.9959	0.9952	0.9943	0.9933	0.9922	0.9910	0.9896	0.9880
11	0.9991	0.9989	0.9986	0.9983	0.9980	0.9976	0.9971	0.9966	0.9960	0.9953
12	0.9997	0.9997	0.9996	0.9995	0.9993	0.9992	0.9990	0.9988	0.9986	0.9983
13	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	0.9996	0.9995	0.9994
14	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$\lambda$	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50
$x = 0$	0.0067	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001
1	0.0404	0.0266	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008
2	0.1247	0.0884	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042
3	0.2650	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149
4	0.4405	0.3575	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403
5	0.6160	0.5289	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885
6	0.7622	0.6860	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649
7	0.8666	0.8095	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687
8	0.9319	0.8944	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918
9	0.9682	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218
10	0.9863	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453
11	0.9945	0.9890	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520
12	0.9980	0.9955	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364
13	0.9993	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981
14	0.9998	0.9994	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400
15	0.9999	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665
16	1.0000	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823
17	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911
18	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957
19	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

### CUMULATIVE POISSON PROBABILITIES

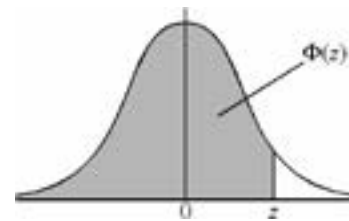
$\lambda$	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00
$x = 0$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000
4	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002	0.0001	0.0000
5	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007	0.0003	0.0002
6	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021	0.0010	0.0005
7	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054	0.0029	0.0015
8	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126	0.0071	0.0039
9	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261	0.0154	0.0089
10	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491	0.0304	0.0183
11	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847	0.0549	0.0347
12	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350	0.0917	0.0606
13	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009	0.1426	0.0984
14	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808	0.2081	0.1497
15	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715	0.2867	0.2148
16	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	0.5660	0.4677	0.3751	0.2920
17	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	0.6593	0.5640	0.4686	0.3784
18	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	0.7423	0.6550	0.5622	0.4695
19	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	0.8122	0.7363	0.6509	0.5606
20	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	0.8682	0.8055	0.7307	0.6472
21	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	0.9108	0.8615	0.7991	0.7255
22	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	0.9418	0.9047	0.8551	0.7931
23	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	0.9633	0.9367	0.8989	0.8490
24	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	0.9777	0.9594	0.9317	0.8933
25	1.0000	0.9999	0.9997	0.9990	0.9974	0.9938	0.9869	0.9748	0.9554	0.9269
26	1.0000	1.0000	0.9999	0.9995	0.9987	0.9967	0.9925	0.9848	0.9718	0.9514
27	1.0000	1.0000	0.9999	0.9998	0.9994	0.9983	0.9959	0.9912	0.9827	0.9687
28	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9978	0.9950	0.9897	0.9805
29	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9973	0.9941	0.9882
30	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9986	0.9967	0.9930
31	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9982	0.9960
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.9978
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9988
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

## THE NORMAL DISTRIBUTION FUNCTION

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $z$ , the table gives the value of  $\Phi(z)$ , where

$$\Phi(z) = P(Z \leq z).$$

For negative values of  $z$  use  $\Phi(-z) = 1 - \Phi(z)$ .



$z$	0	1	2	3	4	5	6	7	8	9	ADD								
	0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

### Critical values for the normal distribution

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $p$ , the table gives the value of  $z$  such that

$$P(Z \leq z) = p.$$

$p$	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$z$	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

### CRITICAL VALUES FOR THE *t*-DISTRIBUTION

If *T* has a *t*-distribution with *ν* degrees of freedom then, for each pair of values of *p* and *ν*, the table gives the value of *t* such that:

$$P(T \leq t) = p.$$



<i>p</i>	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
<i>ν</i> = 1	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291



## Mathematical notation

Examinations for the syllabus in this booklet may use relevant notation from the following list.

### 1 Set notation

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \dots\}$	the set with elements $x_1, x_2, \dots$
$\{x : \dots\}$	the set of all $x$ such that ...
$n(A)$	the number of elements in set $A$
$\emptyset$	the empty set
$\mathcal{E}$	the universal set
$A'$	the complement of the set $A$
$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \dots\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \dots\}$
$\mathbb{Z}_n$	the set of integers modulo $n$ , $\{0, 1, 2, \dots, n - 1\}$
$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
$\mathbb{Q}_0^+$	set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geq 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R} : x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geq 0\}$
$\mathbb{C}$	the set of complex numbers
$(x, y)$	the ordered pair $x, y$
$A \times B$	the cartesian product of sets $A$ and $B$ , i.e. $A \times B = \{(a, b) : a \in A, b \in B\}$
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$
$[a, b)$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
$(a, b)$	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$y R x$	$y$ is related to $x$ by the relation $R$
$y \sim x$	$y$ is equivalent to $x$ , in the context of some equivalence relation

## 2 Miscellaneous symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to
$\infty$	infinity
$p \wedge q$	$p$ and $q$
$p \vee q$	$p$ or $q$ (or both)
$\sim p$	not $p$
$p \Rightarrow q$	$p$ implies $q$ (if $p$ then $q$ )
$p \Leftarrow q$	$p$ is implied by $q$ (if $q$ then $p$ )
$p \Leftrightarrow q$	$p$ implies and is implied by $q$ ( $p$ is equivalent to $q$ )
$\exists$	there exists
$\forall$	for all

## 3 Operations

$a + b$	$a$ plus $b$
$a - b$	$a$ minus $b$
$a \times b, ab, a.b$	$a$ multiplied by $b$
$a \div b, \frac{a}{b}, a / b$	$a$ divided by $b$
$\sum_{i=1}^n a_i$	$a_1 + a_2 + \dots + a_n$
$\prod_{i=1}^n a_i$	$a_1 \times a_2 \times \dots \times a_n$
$\sqrt{a}$	the positive square root of $a$
$ a $	the modulus of $a$
$n!$	$n$ factorial
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n \in \mathbb{Z}^+$ or
	$\frac{n(n-1)\dots(n-r+1)}{r!}$ for $n \in \mathbb{Q}$

## 4 Functions

$f(x)$	the value of the function $f$ at $x$
$f: A \rightarrow B$	$f$ is a function under which each element of set $A$ has an image in set $B$
$f: x \mapsto y$	the function $f$ maps the element $x$ to the element $y$
$f^{-1}$	the inverse function of the function $f$
$gf$	the composite function of $f$ and $g$ which is defined by $gf(x) = g(f(x))$
$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as $x$ tends to $a$

$\Delta x, \delta x$	an increment of $x$
$\frac{dy}{dx}$	the derivative of $y$ with respect to $x$
$\frac{d^n y}{dx^n}$	the $n$ th derivative of $y$ with respect to $x$
$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ..., $n$ th derivatives of $f(x)$ with respect to $x$
$\int y dx$	the indefinite integral of $y$ with respect to $x$
$\int_a^b y dx$	the definite integral of $y$ with respect to $x$ between the limits $x = a$ and $x = b$
$\frac{\partial V}{\partial x}$	the partial derivative of $V$ with respect to $x$
$\dot{x}, \ddot{x}, \dots$	the first, second, ... derivatives of $x$ with respect to $t$

### 5 Exponential and logarithmic functions

$e$	base of natural logarithms
$e^x, \exp x$	exponential function of $x$
$\log_a x$	logarithm to the base $a$ of $x$
$\ln x, \log_e x$	natural logarithm of $x$
$\lg x, \log_{10} x$	logarithm of $x$ to base 10

### 6 Circular and hyperbolic functions

$\left. \begin{array}{l} \sin, \cos, \tan \\ \operatorname{cosec}, \sec, \cot \end{array} \right\}$	the circular functions
$\left. \begin{array}{l} \sin^{-1}, \cos^{-1}, \tan^{-1} \\ \operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1} \end{array} \right\}$	the inverse circular functions
$\left. \begin{array}{l} \sinh, \cosh, \tanh \\ \operatorname{cosech}, \operatorname{sech}, \operatorname{coth} \end{array} \right\}$	the hyperbolic functions
$\left. \begin{array}{l} \sinh^{-1}, \cosh^{-1}, \tanh^{-1} \\ \operatorname{cosech}^{-1}, \operatorname{sech}^{-1}, \operatorname{coth}^{-1} \end{array} \right\}$	the inverse hyperbolic functions

### 7 Complex numbers

$i$	square root of $-1$
$z$	a complex number, $z = x + iy = r(\cos \theta + i \sin \theta)$
$\operatorname{Re} z$	the real part of $z$ , $\operatorname{Re} z = x$
$\operatorname{Im} z$	the imaginary part of $z$ , $\operatorname{Im} z = y$
$ z $	the modulus of $z$ , $ z  = \sqrt{x^2 + y^2}$
$\arg z$	the argument of $z$ , $\arg z = \theta, -\pi < \theta \leq \pi$
$z^*$	the complex conjugate of $z$ , $x - iy$

### 8 Matrices

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
$\mathbf{M}^T$	the transpose of the matrix $\mathbf{M}$
$\det \mathbf{M}$ or $ \mathbf{M} $	the determinant of the square matrix $\mathbf{M}$

**9 Vectors**

<b>a</b>	the vector <b>a</b>
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment $AB$
$\hat{\mathbf{a}}$	a unit vector in the direction of <b>a</b>
<b>i, j, k</b>	unit vectors in the directions of the cartesian coordinate axes
$ \mathbf{a} , a$	the magnitude of <b>a</b>
$ \overrightarrow{AB} , AB$	the magnitude of $\overrightarrow{AB}$
<b>a.b</b>	the scalar product of <b>a</b> and <b>b</b>
<b>a × b</b>	the vector product of <b>a</b> and <b>b</b>

**10 Probability and statistics**

$A, B, C$ , etc.	events
$A \cup B$	union of the events $A$ and $B$
$A \cap B$	intersection of the events $A$ and $B$
$P(A)$	probability of the event $A$
$A'$	complement of the event $A$
$P(A B)$	probability of the event $A$ conditional on the event $B$
$X, Y, R$ , etc.	random variables
$x, y, r$ , etc.	values of the random variables $X, Y, R$ , etc.
$x_1, x_2, \dots$	observations
$f_1, f_2, \dots$	frequencies with which the observations $x_1, x_2, \dots$ occur
$p(x)$	probability function $P(X=x)$ of the discrete random variable $X$
$p_1, p_2, \dots$	probabilities of the values $x_1, x_2, \dots$ of the discrete random variable $X$
$f(x), g(x), \dots$	the value of the probability density function of a continuous random variable $X$
$F(x), G(x), \dots$	the value of the (cumulative) distribution function $P(X \leq x)$ of a continuous random variable $X$
$E(X)$	expectation of the random variable $X$
$E(g(X))$	expectation of $g(X)$
$\text{Var}(X)$	variance of the random variable $X$
$G(t)$	probability generating function for a random variable which takes the values $0, 1, 2, \dots$
$B(n, p)$	binomial distribution with parameters $n$ and $p$
$\text{Geo}(p)$	geometric distribution with parameter $p$
$\text{Po}(\lambda)$	Poisson distribution with parameter $\lambda$
$N(\mu, \sigma^2)$	normal distribution with mean $\mu$ and variance $\sigma^2$
$\mu$	population mean
$\sigma^2$	population variance
$\sigma$	population standard deviation
$\bar{x}, \mathbf{m}$	sample mean
$s^2, \hat{\sigma}^2$	unbiased estimate of population variance from a sample,
	$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$
$\phi$	probability density function of the standardised normal variable with distribution $N(0, 1)$
$\Phi$	corresponding cumulative distribution function

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## Additional information

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### Equality and inclusion

This syllabus complies with our *Code of Practice* and *Ofqual General Conditions of Recognition*.

We have taken great care to avoid bias of any kind in the preparation of this syllabus and related assessment materials. In our effort to comply with the UK Equality Act (2010), we have taken all reasonable steps to avoid direct and indirect discrimination.

The standard assessment arrangements may present barriers for candidates with impairments. Where a candidate is eligible, we may be able to make arrangements to enable that candidate to access arrangements and receive recognition of their attainment. We do not agree access arrangements if they give candidates an unfair advantage over others or if they compromise the standards being assessed. Candidates who are unable to access the assessment of any component may be eligible to receive an award based on the parts of the assessment they have taken. Information on access arrangements is found in the *Cambridge Handbook (UK)*, for the relevant year, which can be downloaded from the website [www.cambridgeinternational.org/eoguide](http://www.cambridgeinternational.org/eoguide)

### Guided learning hours

Cambridge Pre-U syllabuses are designed on the assumption that learners have around 380 guided learning hours per Principal Subject over the duration of the course, but this is for guidance only. The number of hours may vary according to curricular practice and the learners' prior experience of the subject.

### Total qualification time

This syllabus has been designed assuming that the total qualification time per subject will include both guided learning and independent learning activities. The estimated number of guided learning hours for this syllabus is 380 hours over the duration of the course. The total qualification time for this syllabus has been estimated to be approximately 500 hours per subject over the duration of the course. These values are guidance only. The number of hours required to gain the qualification may vary according to local curricular practice and the learners' prior experience of the subject.

### Entries

For entry information, please refer to the *Cambridge Guide to Making Entries (UK)* for the relevant year, available from the website [www.cambridgeinternational.org/eoguide](http://www.cambridgeinternational.org/eoguide)

### If you are not yet a Cambridge school

Learn about the benefits of becoming a Cambridge school at [www.cambridgeinternational.org/join](http://www.cambridgeinternational.org/join)  
Email us at [info@cambridgeinternational.org](mailto:info@cambridgeinternational.org) to find out how your organisation can register to become a Cambridge school.

### Language

This syllabus and the associated assessment materials are available in English only.

